MATH 1325

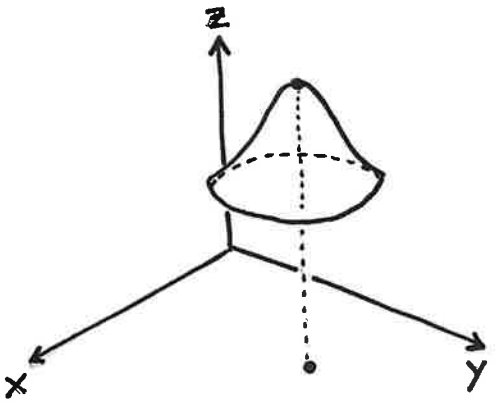
Chapter 14.3: Extrema Of Functions Of Several Variables

Local Maximum and Minimum for (a,b) in $f(x,y)$

Maximum at (a,b)

$f(a,b) \geq f(x,y)$

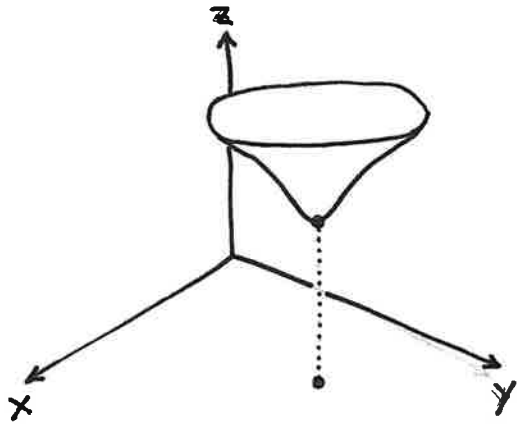
for (x,y) in the (a,b) circular region



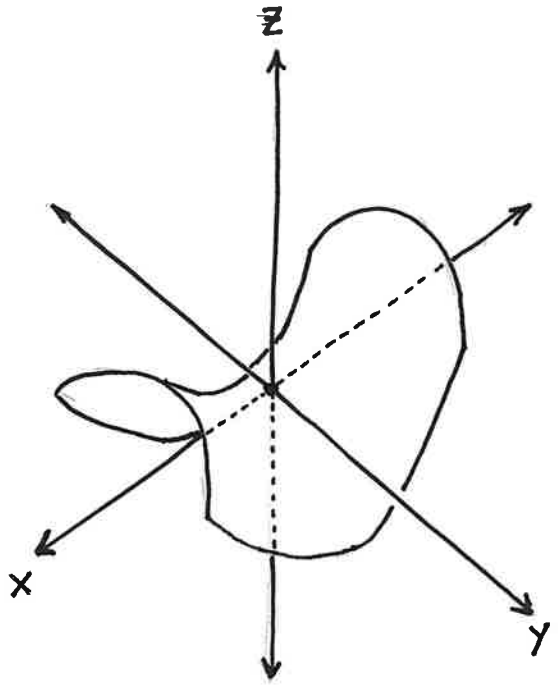
Minimum at (a,b)

$f(a,b) \leq f(x,y)$

for (x,y) in the (a,b) circular region



If $f(x,y)$ has a local maximum or minimum at (a,b) and $f_x(a,b)$ and $f_y(a,b)$ both exist, then $f_x(a,b) = f_y(a,b) = 0$.



"Saddle Point"

$z = f(x,y) = x^2 - y^2$

$f_x(0,0) = 0$ and $f_y(0,0) = 0$

But... $(0,0)$ is not a local maximum or minimum

$(0,0,0)$ is a minimum when approached from the x-direction and it is a maximum when approached from the y-direction

"Critical Point"

$f_x(a,b)$ and $f_y(a,b) = 0$ or DNE.

FIND ALL CRITICAL POINTS FOR $f(x,y) = 6x^2 + 6y^2 + 6xy + 36x - 54y - 5$.

TEST FOR LOCAL EXTREMA:

$f(x,y)$ AND f_{xx}, f_{yy}, AND f_{xy} EXIST.

(a,b) IS A CRITICAL POINT WHERE $f_x(a,b)$ AND $f_y(a,b) = 0$.

$$M = [f_{xx}(a,b) \cdot f_{yy}(a,b)] - [f_{xy}(a,b)]^2$$

M	$f_{xx}(a,b) < 0$	$f_{xx}(a,b) > 0$
$M > 0$	LOCAL MAX	LOCAL MIN
$M = 0$	NO INFORMATION	
$M < 0$	SADDLE POINT	

$f(x,y) = 6x^2+6y^2+6xy+36x-54y-5$ HAS A CRITICAL POINT AT $(-7,8)$.
IS $(-7,8)$ A MAXIMUM, MINIMUM, OR NEITHER?

FIND ALL LOCAL EXTREMA FOR $f(x,y) = 9xy-x^3-y^3-6$.

A COMPANY'S COST TO PRODUCE A BATCH OF A SOFT DRINK IS

$$C(x,y) = 2200 + 27x^3 - 72xy + 8y^2$$

WITH x THE SUGAR $^{kg}/_{BATCH}$ AND y THE FLAVORING $^{g}/_{BATCH}$.

FIND x AND y TO MINIMIZE THE COST. FIND THE MINIMUM COST.