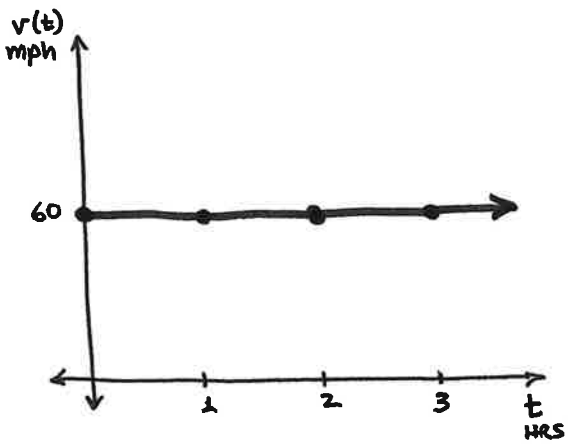
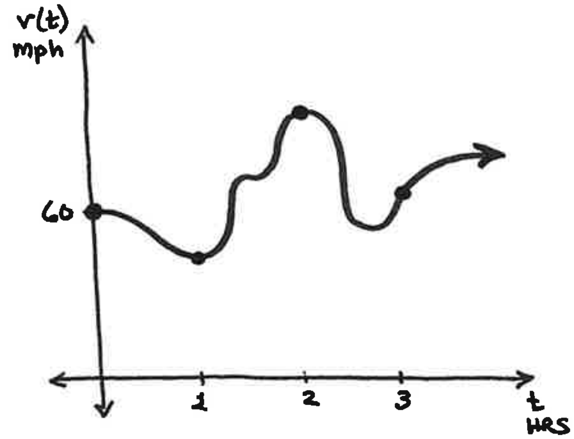
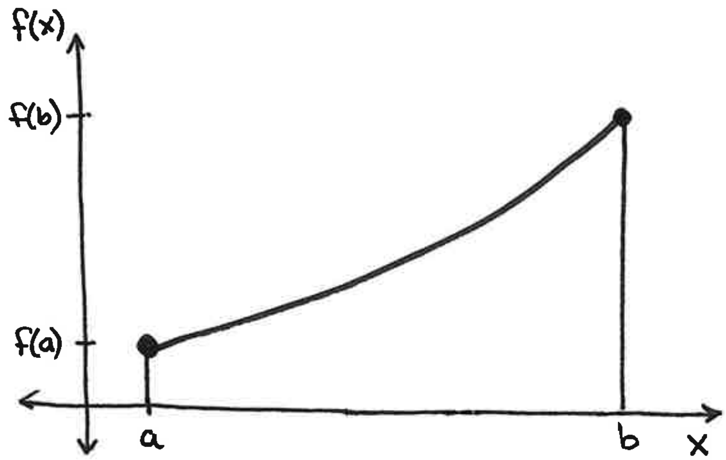
MATH 1325
Chapter 13.4: Area And The Definite Integral

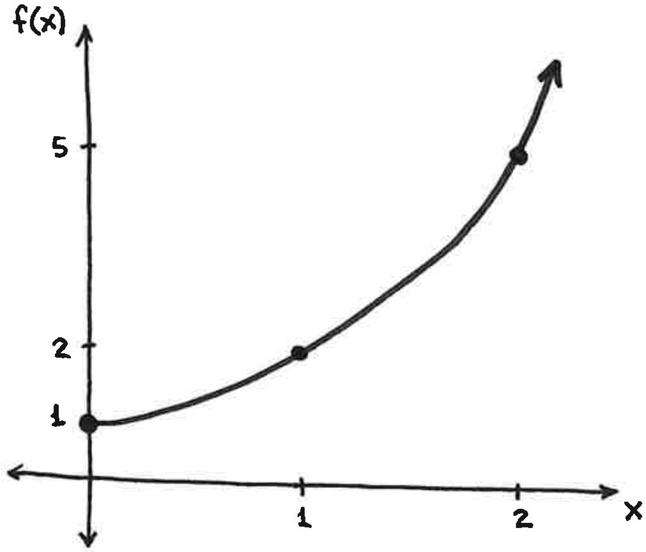




RECTANGULAR APPROXIMATION METHOD (RAM)

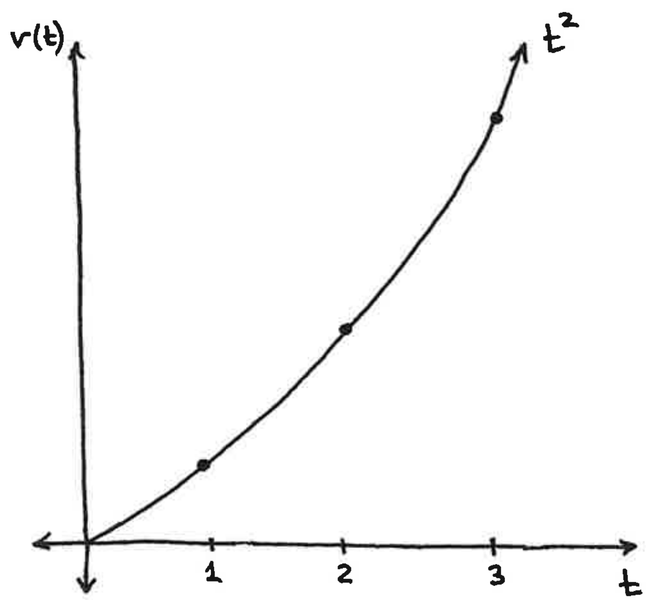


LET $f(x) = x^2 + 1$. FIND THE AREA UNDER THE CURVE FROM 0 TO 2 USING 2 RECTANGLES.



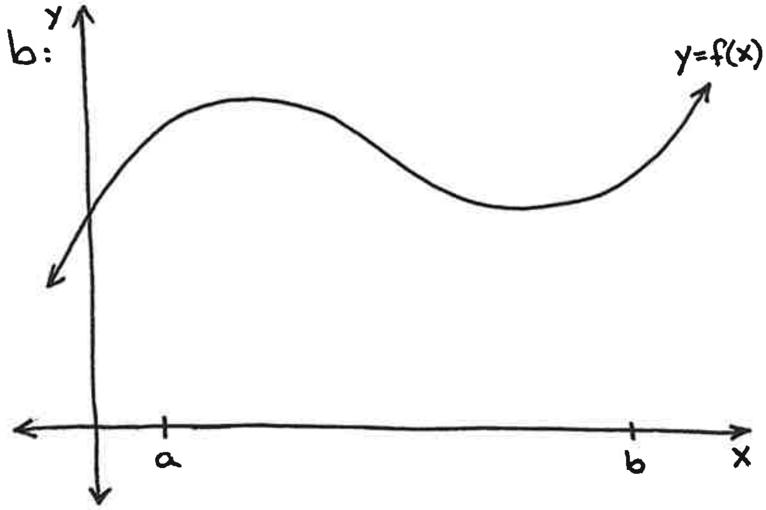
A PARTICLE STARTS AT $x = 0$ AND MOVES ALONG THE X-AXIS WITH VELOCITY $v(t) = t^2$ FOR TIME $t \geq 0$.

WHERE IS THE PARTICLE AT $t = 3$?



DEFINITE INTEGRAL: "AREA UNDER CURVE"

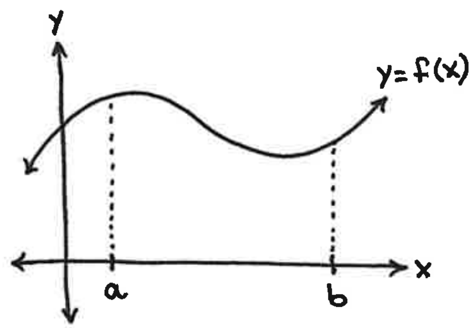
FINDING THE AREA OF $f(x)$ FROM a TO b:



- DIVIDE $[a,b]$ INTO N SUBINTERVALS.
- LENGTH OF SUBINTERVAL: ΔX.
- K^{TH} SUBINTERVAL HAS LENGTH ΔX_K.
- SELECT C_K IN K^{TH} SUBINTERVAL.
- RECTANGLE: WIDTH ΔX_K HEIGHT $f(C_K)$
- AREA OF RECTANGLE: $f(C_K)\cdot \Delta X_K$
- SUM OF ALL RECTANGLES:

$$S_N = \sum_{K=1}^{N} f(C_K)\cdot \Delta X_K$$

BERNHARD RIEMANN (1826-1866)
"RIEMANN SUM"
HE PROVED THE LIMIT EXISTS

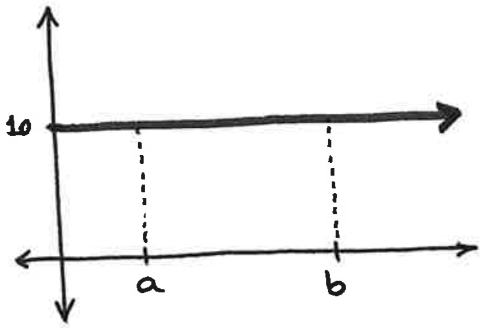


DEFINITE INTEGRAL: IF $f(x)$ IS CONTINUOUS ON $a \le x \le b$, THEN THE DEFINITE INTEGRAL EXISTS

DEFINITE INTEGRAL OF $f(x)$ OVER $[a,b]$ $= \lim_{N \to \infty} \sum_{K=1}^{N} f(C_K)\cdot \Delta X$

AREA UNDER $f(x)$ FROM a TO b $=$

CONSIDER $f(x) = 10$.



"AREA UNDER CURVE"

$$A = \int_a^b f(x)\,dx$$

APPROXIMATE $\int_1^4 4x\,dx$ USING SIX RECTANGLES.